Problem 14 Ice Clock

Vivian Gao, Shantanu Kadam
String Bean Theorists
UC Berkeley

Prompt, Key Parameters

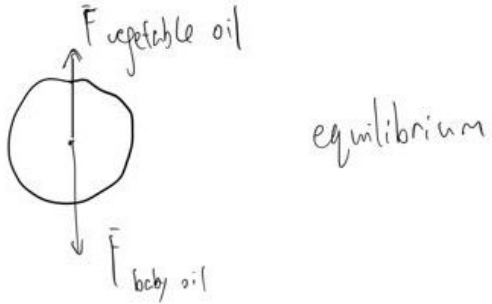
An ice cube inside a mix of vegetable oil and baby oil will remain between the two liquids because of its density. As the ice cube melts and releases trapped bubbles, it goes up and down periodically in an intriguing way.

1. Can this experiment be turned into a clock? What would be its longevity and precision?
    a. Longevity ~ duration in seconds, precision ~ period
2. Optimize the setup parameters (shape, temperature, composition, length scales, etc.) to obtain the **maximal clock precision.**
    a. 60 seconds

Qualitative Theory - cycle

- First drop the ice - equilibrium (In between two oil)

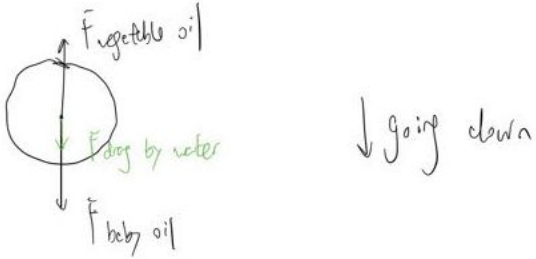
- Vegetable oil density: ~ 0.93 g/mL
- Baby oil density: ~ 0.83 g/mL
- Ice density: 0.9 g/mL

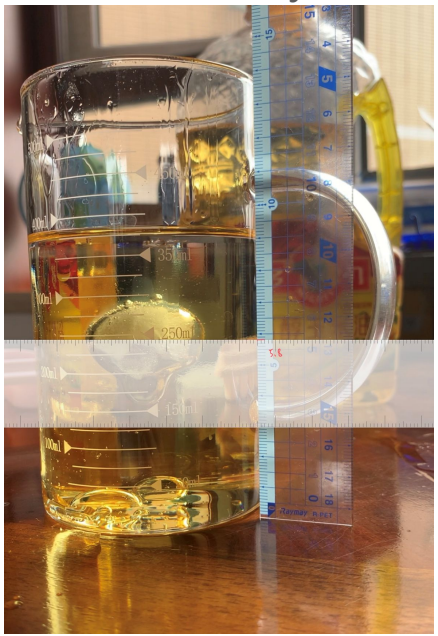


Prompt Theory Experiment Analysis Results

Qualitative Theory - cycle

- Temperature difference---ice melts - water density>vegetable oil density ----> sink
- Water is contacting with the cube, so dragging the cube down together with the downwards force provided by baby oil

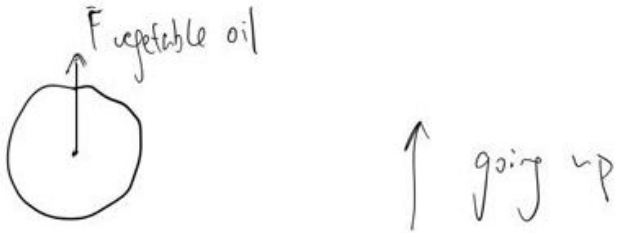


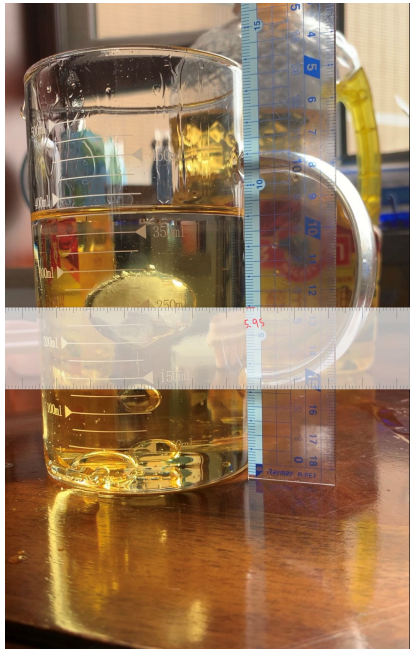


Prompt Theory Experiment Analysis Results

Qualitative Theory - cycle

- Water leaves the ice, and the ice floats up as it is now surrounded by vegetable oil

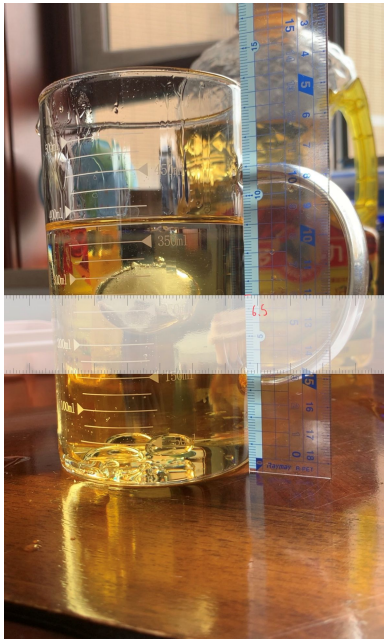


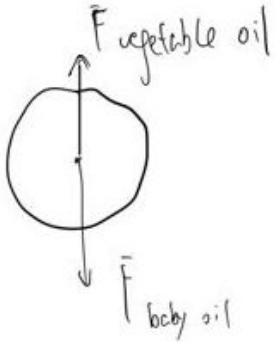


Prompt Theory Experiment Analysis Results

Qualitative Theory - cycle

- Cube returns to its original position (In between two oil)





equilibrium

Prompt Theory Experiment Analysis Results

Quantitative Theory: Harmonic Oscillation?

- Cycling ⇒ oscillation
  - Simple Harmonic Oscillation for a single cycle:

$$d(t,\omega) = A\sin(\omega t)$$

$$f = \frac{\omega}{2\pi}, T = \frac{1}{f} \Rightarrow \omega = \frac{2\pi}{T}$$

$$d(t,T) = A\sin\left(\frac{2\pi}{T}t\right)$$

  - Maximal precision: $T = 60$

What does the theory tell us about optimal setups?

- Parameters we focused on:
  - Shape
  - Temperature
  - Composition

Expectation when changing temperature

- Oscillation will be faster as ice melts faster

Expectation when changing shape

- The time when water and ice is getting contact with each other may change as the shape changes, because they are having different configuration
  - The sharper the corner is, the quicker the water drop will leave the ice
    - It’s harder to maintain the drop at a sharper corner, as it follows the tendency to drop

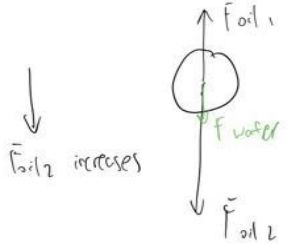
Expectation when changing composition

- Both oil have density <0.9 : SINK

- Both oil have density >0.9: Float at the surface of the liquid, and the cube stays still on the surface while dropping the water down. (There is no downwards force in the oil)

- Oil 1 with 0.9<density<1, oil 2 with density < 0.9
  - Gets complicated if wants to maximize displacement

oil 2 as small as possible



$\therefore$ travel more distance

$F_{oil\,2}$ increases

Cannot be really small

oil 1 $\longrightarrow$ 1

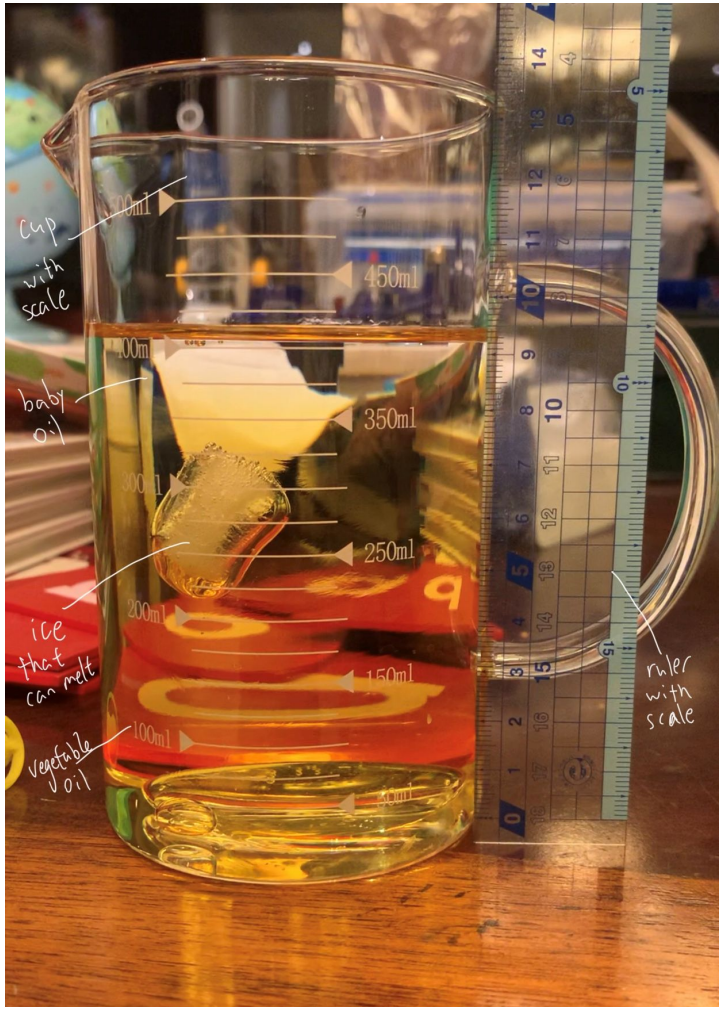
water decreases

$F_{oil\,1}$ increases

If gets too close to 1, oscillation may not happen

Experimental setup

- Same setup to answer both questions
  - Use as clock
  - Optimal setup



Prompt

Theory

Experiment

Analysis

Results

Qualitative factor - temperature

Prompt Theory Experiment Analysis Results

Qualitative factor - shape

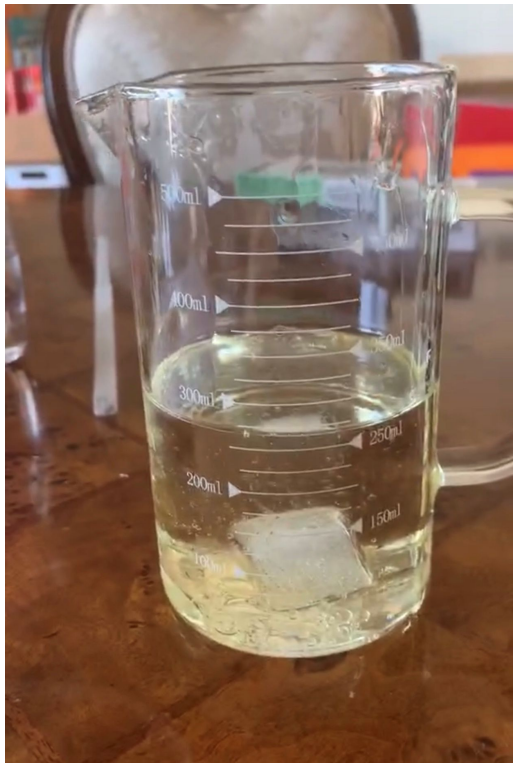
Trapezium

Cube

Sphere

Qualitative factor - composition

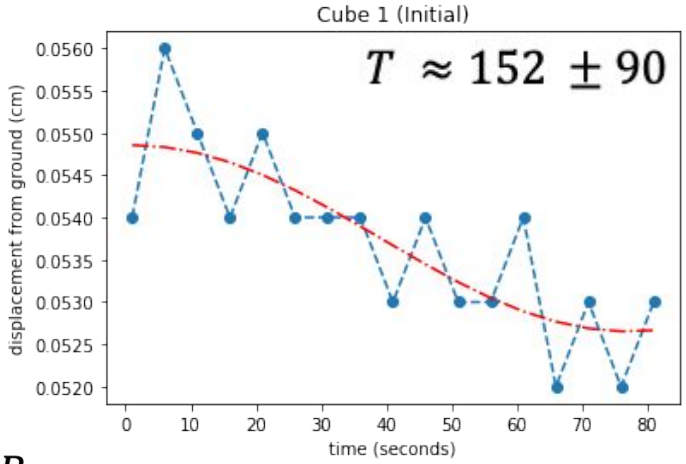
- Olive oil ~ 0.88 g/ml
- Baby oil ~ 0.83 g/ml

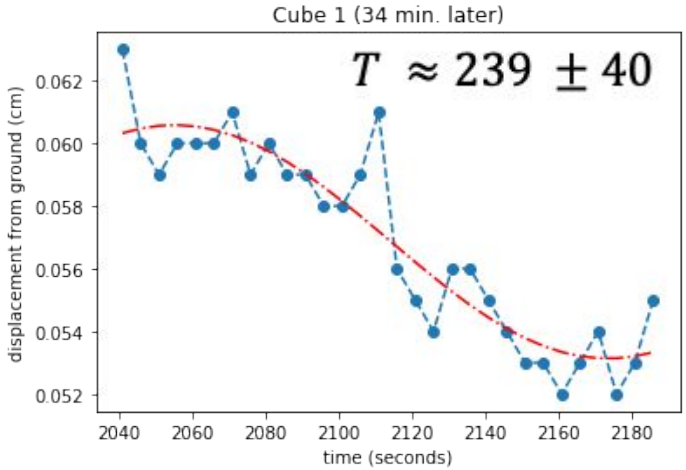


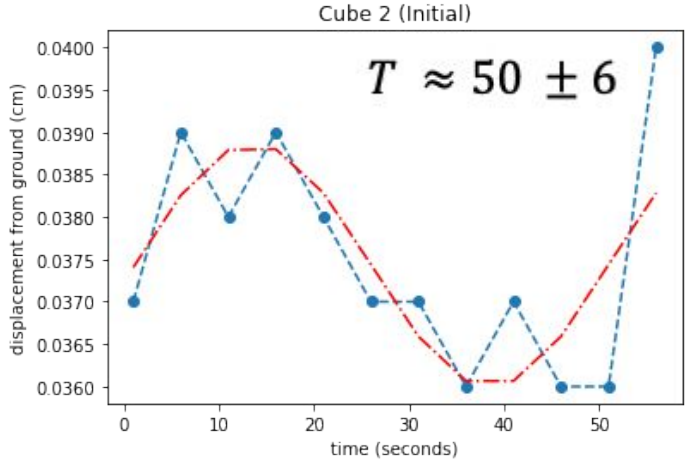
- Vegetable oil ~ 0.93 g/ml

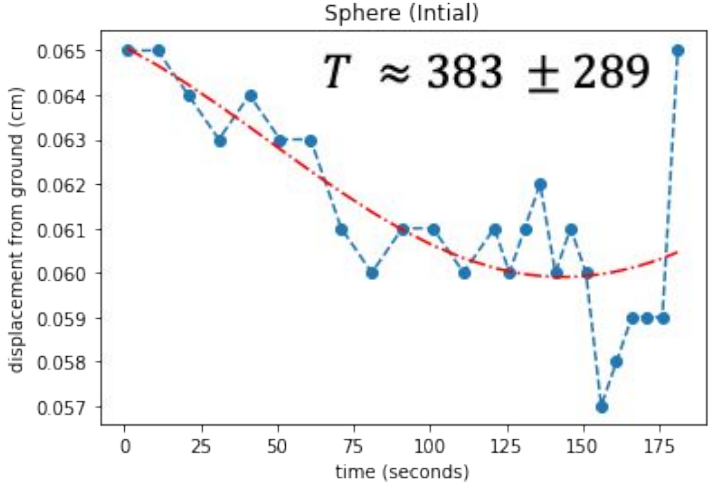
Data Analysis

$$d(t,T) = A\sin\left(\frac{2\pi}{T}t - C\right) + B$$







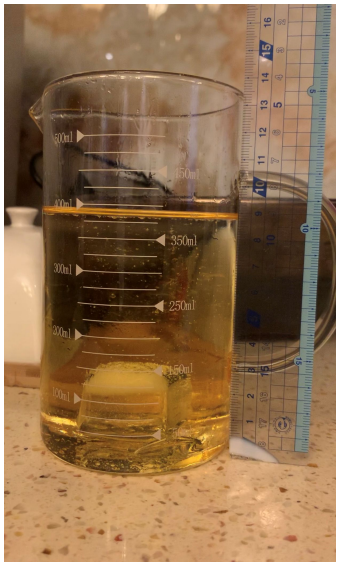


Results: Usage as Clock

- This phenomenon can be turned into a clock.
- Precision: can tune setup to drop water after a minute
- Longevity: did not time, but can run for over 36 minutes (Cube 1)

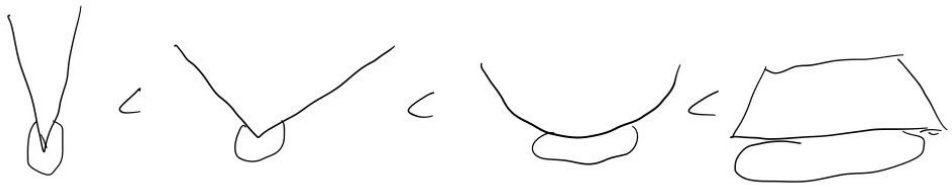
Results: Optimal Setup (temperature)

- The experiment failed because as temperature increases, the density of the oil decreases, so the vegetable oil drops to < 0.9 g/ml
  - The ice simply sank to the bottom



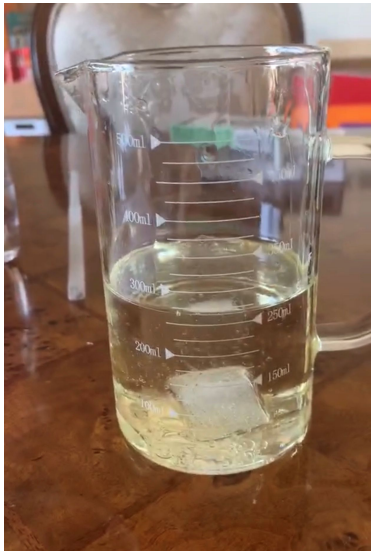
Results: Optimal Setup (shape)

- 57 seconds - cube - one cycle
- 186 seconds - spherical ball - one cycle
- 177 seconds - trapezium - less than half of a cycle (it sunk to the bottom at the end)



Results: Optimal Setup (composition)

- Olive oil ~ 0.88 g/ml
- Baby oil ~ 0.83 g/ml
- SINK



- Vegetable oil ~ 0.93 g/ml

Conclusion: Optimal Setup for Maximal Precision

- Use ice cube ~ 57 seconds at 21.7 degree celsius at the beginning of the experiment
    - Increase temperature by a tiny amount --- not big enough for the density of the oil to decrease

Future consideration

1. Use as Clock
    - Update experimental setup: stabilize phone ⇒ increase resolution, precision
    - Collect data over multiple cycles ⇒ precision of clock and repeated longevity
        - Damped oscillation?
2. Optimal Setup
    - Have more changes on the composition of oil (consider third expectation experimentally to verify the theory)
        - Oil 1 with 0.9<density<1, oil 2 with density < 0.9
    - Triangular ice, to see its period, and verify the theory experimentally for shape changes
    - Maybe try to test the amount of increase in temperature for the setup for cube, and obtain 60 seconds period.

References

- *Computational Physics* by Mark Newman

Curve Fitting Results

Sphere

amplitude=-0.003+/-0.003

period=382.573+/-288.503

offset vertical= 0.063+/-0.003

offset horizontal= 0.786+/-1.613

Cube 2

amplitude= 0.001+/-0.000
period=49.815+/-5.674
offset vertical= 0.037+/-0.000
offset horizontal= 6.427+/-0.495

Cube 1 (34 min.)

amplitude=-0.004+/-0.000
period=236.854+/-39.831
offset vertical= 0.057+/-0.000
offset horizontal=37.241+/-9.424

Cube 1

amplitude=-0.001+/-0.000
period=152.319+/-90.227
offset vertical= 0.054+/-0.000
offset horizontal= 1.615+/-1.134