SPS Problem of the Week 11/7/2020-11/14/2020

Problem 1. Twenty-three people, each with integral weight, decide to play football, separating into two teams of 11 people, plus a referee. To keep things fair, the teams chosen must have equal total weight. It turns out that no matter who is chosen to be the referee, this can always be done. Prove that all 23 people must have the same weight.