SPS Problem of the Week 10/31/2020-11/7/2020

Problem 1. A **permutation** of the integers $\{1, 2, ..., n\}$ is a rearrangement of these numbers (or more formally a bijection from the set to itself).
For instance one permutation of $1, 2, 3$ is:

$$P : \begin{cases} 1 \to 2 \\ 2 \to 3 \\ 3 \to 1 \end{cases}$$

The permutation P^2 is constructed by applying P on the output of P. Thus in particular,

$$P^2 : \begin{cases} 1 \to 3 \\ 2 \to 1 \\ 3 \to 2 \end{cases}$$

P^3 is similarly P applied to the output of P^2 (you can generalise this to P^k). In our case it just so happens that:

$$P^3 : \begin{cases} 1 \to 1 \\ 2 \to 2 \\ 3 \to 3 \end{cases}$$

Which is the same as the original arrangement.
This prompts the **very interesting question** that if I have any permutation P on the n numbers $\{1, 2, ..., n\}$, does some power P^k of P return the original arrangement?

Hint: This problem is closely related to the fact that if n_7 denotes the remainder of an integer n when divided by 7. Then for any number n there is an integer $k < 7$ such that $k \cdot n = 1_7$.