

SPS Solution of the Week 10/10/2020-10/17/2020

Problem 1. Suppose you and I play the following game. On a rectangular table we start placing pennies one by one such that any new penny placed doesn't touch any penny previously placed. Eventually no more pennies can be added, and the person who places the last valid penny wins.

The question is: Who has to go first for you to have a winning strategy? (And what is this strategy?)

Solution: Notice that intuitively our strategy must adapt to anything our opponent can do, thus the cleanest way to do this would be to exploit the symmetries of our objects.

The pennies and the table share all the symmetries of the table.

A first guess at a strategy might be to mirror about the central vertical line, everything our opponent does, however this does not work since any penny placed on the line itself blocks its image.

A little more playing around leads us to discover that this can be avoided by exploiting reflection about the center (or reflecting about the vertical and horizontal axes simultaneously) since this transformation only has one fixed point (point that is its own image), the center.

Now clearly, the first player can place a coin at the center at the start and then mirror about the center everything their opponent does, to win. \square