SPS Solution of the Week 10/3/2020-10/10/2020

Problem 1. Lazy Sketching

You may have at some point tried to draw the following figure without lifting your pen off of the paper or double crossing any edges.

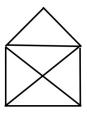


Figure 1:

And if you ever wondered what was so special about this shape, or whether there was anything special at all, which let you draw it this way. You might wonder whether you can draw the following shape in the same manner.

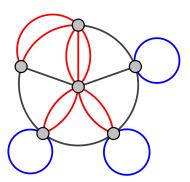


Figure 2:

What you must find out is what properties must a shape made in this way (bunch of points connected by line curves) need in order that it can be drawn without lifting your pen off of the paper or double crossing any of the lines. You can double through vertices of course.

Solution: Everything has a start and an end, and sometimes the start is the same as the end. That is, there can only be 2 or 0 vertices with an odd number of edges attached to it (self loops count twice), for if something is a middle there is a way out for every way in :) \Box